

BEAT CEPHEIDS AS PROBES OF STELLAR AND GALACTIC METALLICITY: II. OPACITIES WITH THE AGS MIXTURE

J. ROBERT BUCHLER¹

Submitted to ApJ

ABSTRACT

It is well known that the mere location of a Beat Cepheid model in a Period Ratio vs. Period diagram (Petersen diagram) puts constraints on its metallicity Z . But these bounds are sensitive to the mixture of elements that are lumped into the parameter Z . In this short paper we update the previous results that were based on the Grevesse-Noels solar mixture to the recent, revised Asplund, Grevesse & Sauval (2005) (AGS) solar mixture. We also examine the effect of the envelope depth on the accuracy of the computed pulsation periods. We find that for low period Cepheids with high Z the customary approximation of envelope pulsation breaks down. It is necessary to compute stellar models that extend to the center and to include burning and composition inhomogeneities in the modeling. Fortunately, however, most Beat Cepheids that have been observed so far seem to avoid that regime.

Subject headings: (stars: variables:) Cepheids, stars: oscillations, stars: rotation, galaxies: abundances

1. INTRODUCTION

A recent paper by Buchler & Szabó (2007) (Paper I, hereafter) described the results of a modeling survey of Beat Cepheids that pulsate in the fundamental (F) mode and the first overtone (O1) simultaneously. In order to allow a convenient comparison with the observations, Paper I presented the results in a P_{10} vs. P_0 diagram, a so called Petersen diagram, as a function of metallicity Z , where P_0 and P_1 are the fundamental and first overtone periods, respectively, and $P_{10} = P_1/P_0$. The two periods can easily be extracted from the observed light curves. From the metallicity dependent Petersen diagram one can set bounds on the metallicity Z of the star. This way of determining the metallicity is independent of and complementary to all other stellar and galactic metallicity probes.

Buchler & Szabó (2007) found that the Petersen diagrams are rather insensitive to stellar rotation rates and to the galactic helium enrichment function $Y = Y(Z)$ that they used. However, the results exhibited sensitivity to the chemical mixture that is lumped into the metallicity parameter Z , mainly through their effect on the opacities. The survey of paper I used the OPAL GN93 opacities (Iglesias & Rogers (1996) for the solar mixture of Grevesse & Noels (1993), merged with the Alexander & Ferguson (1994) opacities at the lower temperatures ($\log T < 3.95$).

In the last couple of years Asplund, Grevesse & Sauval (2005) have dramatically revised the solar chemical composition of metals. The most important change is a decrease of elements with atomic numbers $Z < 11$, but dominated by O, in favor of elements with $Z > 11$ of which Fe has the most important effect on the opacities. In view of the sensitivity of the Petersen diagrams to the chemical mixture we have thought it useful to update our survey with OPAL opacities that reflect these new abundances.

In this study, as in Paper I, we use a solar elemental mixture even in low metallicity Cepheids out of expediency, because there is no observationally derived information available that would let one improve much upon such an approximation at this time.

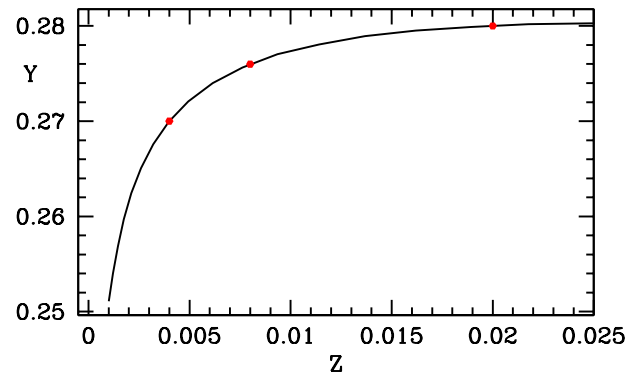


FIG. 1.— Helium galactic enrichment function $Y(Z)$ that is used in the model computations.

In a linear study such as this the Beat Cepheids are defined as being simultaneously unstable in the fundamental (F) and the first overtone (O1) modes. However, it is well known from nonlinear Cepheid studies (*e.g.*, Kolláth et al. (2002); Kolláth, Beaulieu, Buchler, Yecko (1998)) that not all stars in this 'linear Beat Cepheid instability strip' will actually undergo stable Beat pulsations (a large fraction will be either F or O1 Cepheids). This computationally convenient, linear definition therefore considerably overestimates the 'Beat Cepheid instability strip' (broader by at least a factor of 3 – 4 in $\log T_{\text{eff}}$ in a theoretical HR diagram (Szabó & Buchler 2008)). This in turn increases considerably the separation of the upper and lower boundaries in the Petersen diagrams with a concomitant increase in the uncertainty of the stellar metallicities. The linear properties of stars do not allow one to infer the modal selection which is intrinsically nonlinear (Stellingwerf 1974; Buchler & Kovács 1986). Therefore, for a tighter, and correct, definition and study of Beat Cepheids it is necessary to resort to a full amplitude hydrodynamic survey of models which requires a huge computing effort. Such a survey will be presented in a separate paper (Szabó & Buchler (2008)).

In §2 we display the computed Petersen diagrams for the AGS solar elemental composition. In §3 we discuss the ad-

¹ Physics Department, University of Florida, Gainesville, FL 32611, USA

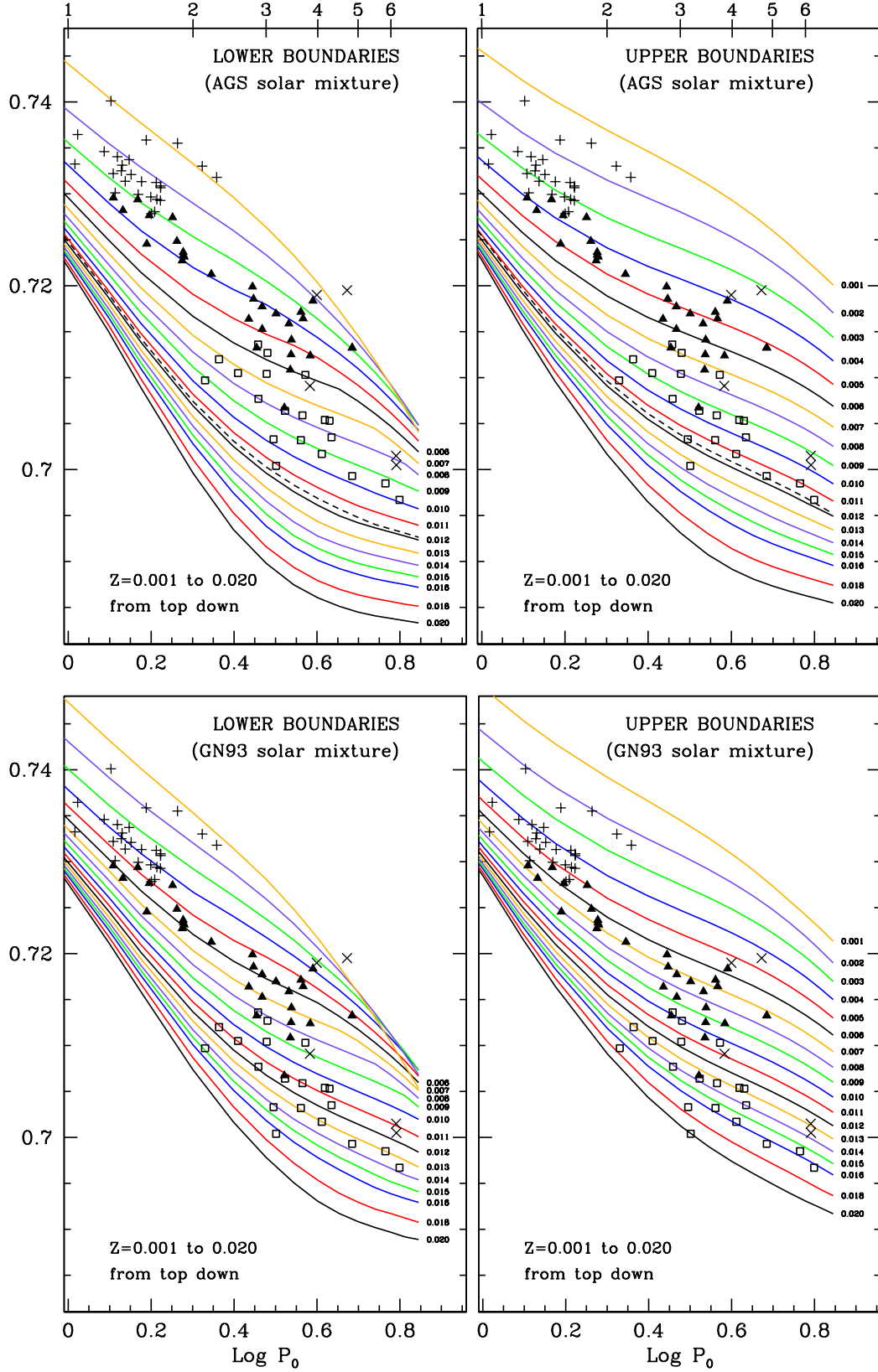


FIG. 2.— P_1/P_0 vs. $\log P_0$ (P_0 on top axis) where P_0 and P_1 are the fundamental and first overtone periods. The lines delimit the ranges for which both F and O1 are linearly unstable: Lower boundaries in the left, and upper boundaries in the right panels. The metallicity increases downward in the figure from $Z = 0.001$ (top line) to 0.016 in steps of 0.001, and then from in steps of 0.002 to 0.020 (bottom line). (In the left panels the lowest Z values are not shown to avoid overcrowding). The long-dashed line refers to the specific AGS composition of the Sun, $Z_{\odot} = 0.0122$ and $Y_{\odot} = 0.2486$. For reference we have superposed the location of most known Beat Cepheids.

TABLE 1
GALACTIC METALLICITIES DERIVED FROM THE BEAT
CEPHEIDS FOR THE AGS AND GN93 ELEMENTAL MIXTURES.

Z_{\min} and Z_{\max} represent the lowest and largest Z values that are obtained for all Beat Cepheids in the respective galaxy. $\langle Z \rangle$ is the average of the centroids of the minimum and the maximum Z of each star in the galaxy.

galaxy	$\langle Z \rangle$	Z_{\min}	Z_{\max}
AGS elemental mixture.			
Galaxy	0.0088	0.0055	0.0133
M33	0.0056	0.0000	0.0097
LMC	0.0046	0.0024	0.0093
SMC	0.0027	0.0010	0.0039
GN93 elemental mixture.			
Galaxy	0.0118	0.0073	0.0182
M33	0.0075	0.0008	0.0124
LMC	0.0062	0.0035	0.0124
SMC	0.0040	0.0016	0.0069

equacy of envelope models for Cepheids, and find that it is necessary to compute full stellar models for the short period, low metallicity Cepheids. We conclude in §4.

2. PETERSEN DIAGRAMS WITH THE AGS CHEMICAL COMPOSITION

This survey proceeds in parallel with the one presented by Buchler & Szabó (2007) for the GN93 solar chemical mixture, and we do not repeat a description of the procedure and of the modeling details here. We just mention that we have kept the same helium enrichment function $Y(Z)$ (Fig 1) that was based on an interpolation between the 'standard' values for the Galaxy and the two Magellanic Clouds. We stress again that the results are very broadly insensitive to this relation. We have used the OPAL website to generate opacities as a function of metallicity Z with the mixture of elements that are taken from the AGS table of solar abundances.

Our newly computed Petersen diagrams for the AGS mixture are displayed in the top panels of Fig. 2. The left-side and right-side panels show the lower and upper bounds, respectively, of the allowed Beat Cepheid regions as a function of metallicity Z . We alternate the line-type (or color) for the successive Z values to make the diagrams easier to use. In order to avoid crowding we have omitted the labels 0.001 to 0.005 on the upper curves in the left-side panels.

The dashed curves in the top panels in Fig. 2 show the results for the opacity with the specific AGS composition for the Sun, viz. $Z_{\odot} = 0.0122$ and $Y_{\odot} = 0.2486$. Despite the large difference in Y (this point lies below the range of the diagram in Fig. 1), this curve falls close to the 0.012 lines, illustrating again the insensitivity to the exact values of Y .

For an easy comparison we have also juxtaposed the GN93 composite Petersen diagram of Paper I in the bottom panels of Fig. 2. The GN93 models have been recomputed with the same zoning (200 mass-shells) as for the new AGS results, which is a little finer than that used in Buchler & Szabó (2007). It is noteworthy that the AGS mixture causes an overall downward shift of the curves, especially for the shorter periods. and a downward stretching for the longer periods. One therefore obtains generally lower metallicities with the AGS solar mixture than with the GN93 solar mixture. This is of course expected since AGS have revised the metallicity for the Sun down to $Z_{\odot} = 0.0122$.

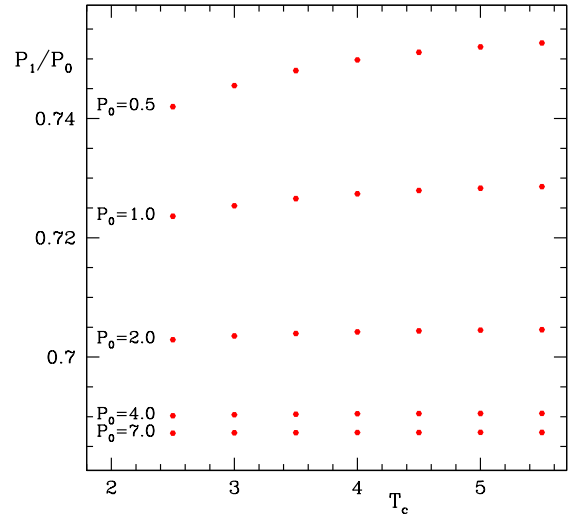


FIG. 3.— The period ratio P_{10} as a function of the temperature of the envelope depth, T_c (in 10^6 K), for the sequences of models of Table 1 with the indicated fundamental periods (in days).

The locations of the known Beat Cepheids are superposed in Fig. 2 as in Buchler & Szabó (2007), with open squares for the Galaxy, x's for M33, filled triangles for the LMC and crosses for the SMC. We have limited the figure to periods longer than one day despite the fact that a few such stars are known in the SMC. The reason will be given in §3.

The diagrams of Fig. 2 are used as follows: If a Beat Cepheid falls between the upper and lower boundaries for a given Z value, that means that models with that value of Z are compatible with the observed period constraints for the given Beat Cepheid. By finding all the compatible Z values one can set upper and lower bounds on Z . In practice, one interpolates between the displayed curves in order to refine the Z bounds.

In Table 1 we present the metallicities for the 4 galaxies in which a significant number of Beat Cepheids are known. From the composite Petersen diagram we infer for each star (i) the range of possible Z , i.e. $Z_{\min,i}$ and $Z_{\max,i}$ which, as we have seen, represent the uncertainty range of Z for each Beat Cepheid (due to the width of the instability strip). We define Z_{\min} as the minimum over the galactic sample of these $Z_{\min,i}$ and Z_{\max} as the maximum of the $Z_{\max,i}$. Finally we introduce the galactic average $\langle Z \rangle$ as the average of the centroids of the ranges $\frac{1}{2}(Z_{\min,i} + Z_{\max,i})$.

With the new opacities we obtain an average metallicity of $\langle Z \rangle = 0.0088$ and a range of uncertainty of 0.0055 to 0.0133 for the Galactic Beat Cepheids. This is to be compared with an average $\langle Z \rangle = 0.0118$ and a range of 0.0073 to 0.0182 with the GN93 opacities. As noted in Paper I the GN93 average $\langle Z \rangle$ from the Galactic Beat Cepheids is quite low compared to the GN93 value for the Sun $Z_{\odot} = 0.017$, the but the AGS the average $\langle Z \rangle$ of the Beat Cepheids is now much closer to the AGS value of $Z_{\odot} = 0.0122$.

We note again here that the large, indicated spread Z_{\min} to Z_{\max} represents a combination of the actual spread in the galactic metallicity, and of the uncertainty that arises from the use of the linear definition of Beat Cepheids. As mentioned earlier, we hope to reduce the latter with a survey of full amplitude Beat Cepheid models Szabó & Buchler (2008).

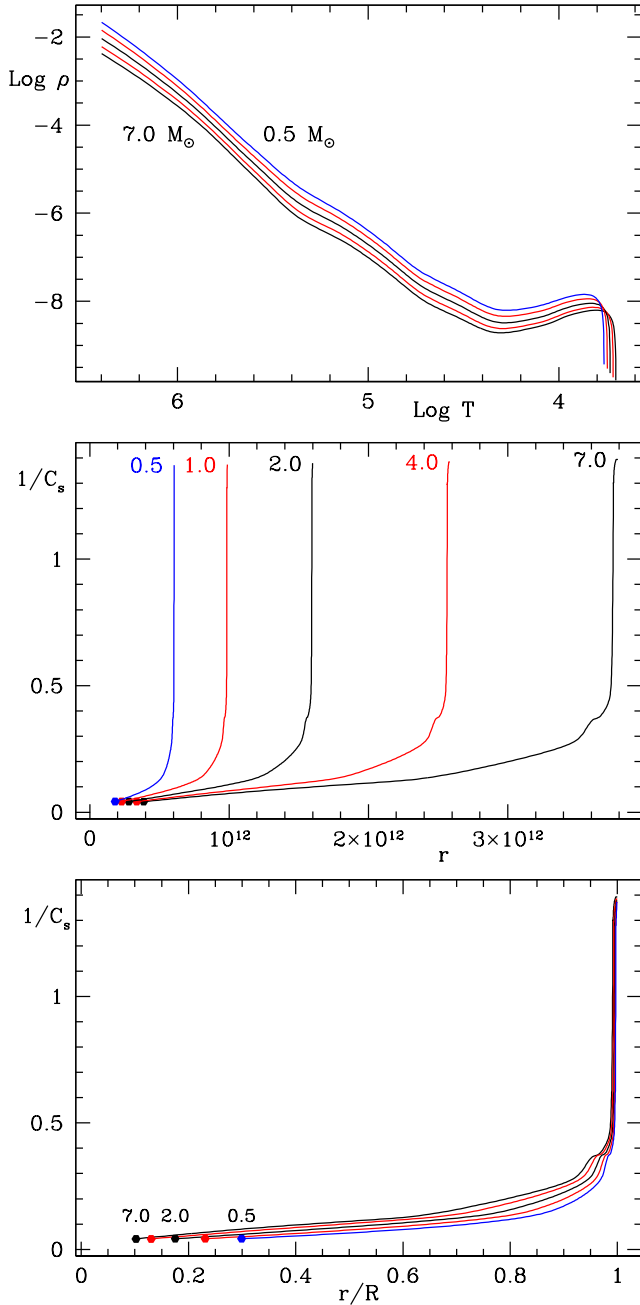


FIG. 4.— (a) Model structure, $\log \rho$ [g/cm^3] vs. $\log T$ [K] for the 5 models of Table 2 on the fundamental blue edge with fundamental periods, 0.5, 1.0, 2.0, 4.0 and 7.0 d; (b) Inverse sound speed (in units of 10^6 cm/s) vs. radius [cm]; (c) Inverse sound speed vs. relative radius.

3. DISCUSSION OF THE MODELING UNCERTAINTIES — INADEQUACY OF ENVELOPE MODELS

Ever since the early days of Cepheid modeling, it has been customary to approximate Cepheids with envelope models of uniform composition, because the pulsation is largely confined to the outer parts of the stars. One thus imposes a rigid core of radius R_c at some temperature T_c , located outside the burning region where the luminosity L_c is constant. This approximation has been justified on the one hand, because the radial displacement eigenvectors for the low lying pulsation modes decay rapidly inward and one therefore sets $\delta r = 0$ at $r = R_c$, and, on the other hand, because the computed peri-

TABLE 2
MODEL SEQUENCES USED IN THE TESTS OF §3

P_0 [d]	M/M_\odot	L/L_\odot	T_{eff} [K]	R_* [10^{11} cm]
0.5	3.09	154	6882	6.0821
1.0	3.63	347	6612	9.8743
2.0	4.32	805	6407	16.0227
4.0	5.14	1806	6175	25.8364
7.0	5.93	3422	5982	37.8964

ods are found not to change much when one varies the depth of the envelope models. However, for the Petersen diagrams we require a higher relative accuracy (< 0.001) in the periods, as we now discuss.

A reexamination of this assumption shows a breakdown for models with low periods and high Z . It turns out that in this regime the periods, and concomitantly the period ratios, exhibit an uncomfortable dependence on R_c or T_c . Furthermore, for the lowest periods there is a poor leveling off even with T_c up to (perhaps unreasonably large) values of 5.5×10^6 K which is beyond the temperature up to which one can neglect nuclear burning and spatial composition inhomogeneities.

This T_c dependence is illustrated in Fig. 3, which plots the period ratios P_{10} vs. T_c for a sequence of 5 models (see Table 2) that have been chosen with a composition of $X = 0.7045$ and $Z = 0.016$ and that are all located at the high temperature side of the region where both the fundamental and the first overtone modes are linearly unstable. The fundamental periods of the sequences are $P_0 = 0.5, 1.0, 2.0, 4.0$, and 7.0 d, respectively. The shape for the leveling off is found to be the same for all sequences, but while the variation in P_{10} remains within an acceptable range for the long period models, this is not the case for the short period models. For example, in the 4 d sequence a change from $T_c = 2.5 \times 10^6$ to 3.0×10^6 K leads to an increase of P_{10} of less than 0.0002, but for the 0.5 d sequence the same change results in an increase of P_{10} of 0.0035. This is a large amount if one considers the Petersen diagram of Fig. 2. Furthermore, for the low period models the leveling off is not complete even by the time one reaches $T_c = 5.5 \times 10^6$ K.

In Fig. 4 we present some other relevant properties of the sequence of models from Table 2. Thus the top panel displays the models in the $\log \rho$ vs. $\log T$ plane, and reveals a very similar structure for the 5 models and demonstrates that nothing special is occurring for the 0.5 d models.

The middle (b) and bottom (c) panels exhibit the inverse of the sound speed vs. radius, r , and vs. relative radius (r/R_*), respectively, where R_* is the equilibrium stellar radius. The dots correspond to the location of the rigid inner core R_c , defined to lie at $T_c = 2.5 \times 10^6$ K in these models. In actual radius, the low period models are much deeper, but this is not the case in relative radius. The reason for presenting these quantities is that the fundamental period P_0 is roughly proportional to the stellar sound traversal time

$$P_0 \propto \int \frac{1}{c_s} dr.$$

The combined effect of the high sound speed in the stellar core and the decay to zero of the eigenvectors cause the contribution from the neglected penetration of the core to be very small in general. It is clear from Fig. 4(b) that the contribution to the period from a deepening of the envelope by

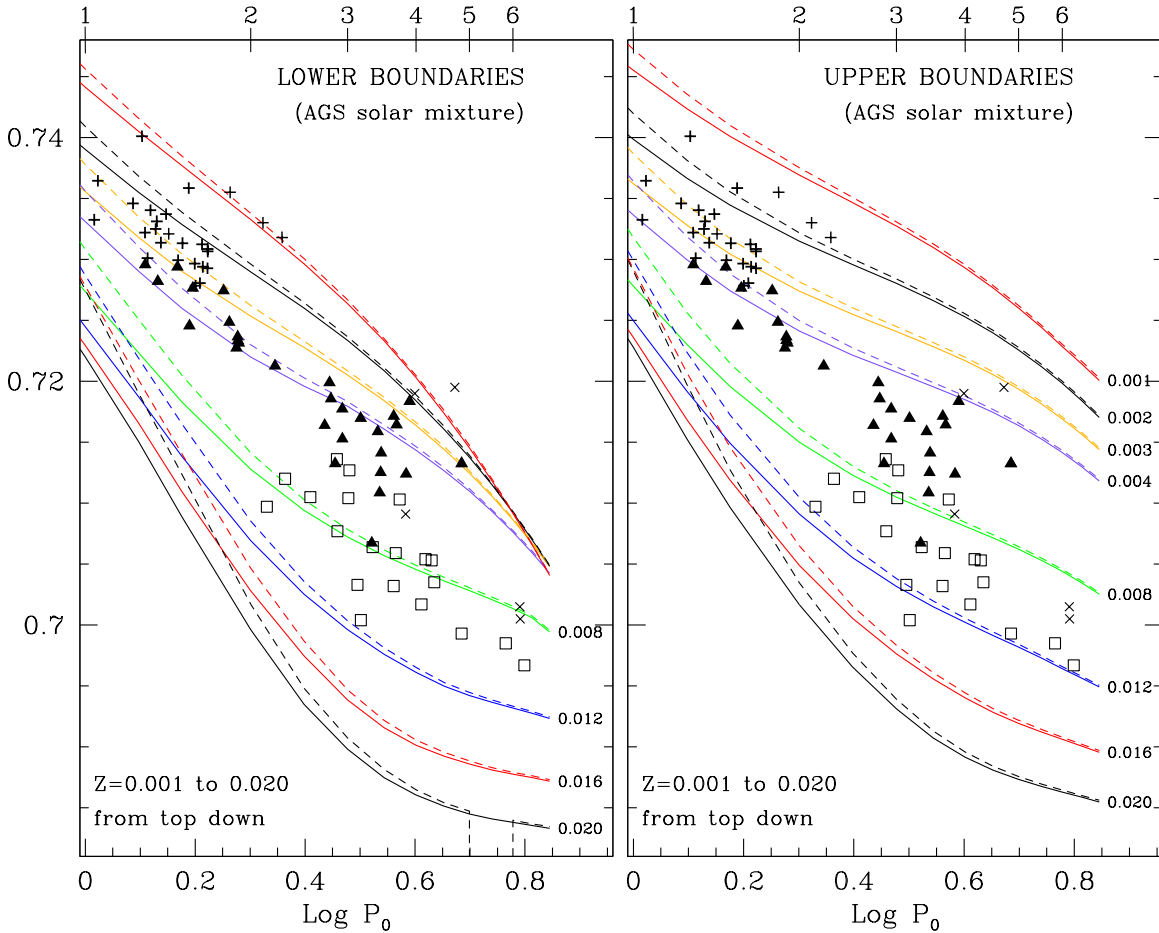


FIG. 5.— Sensitivity to the depth of the envelope T_c of the P_1/P_0 vs. $\log P_0$ [d] Petersen diagrams (for an AGS mixture). The solid lines show the results for an envelope of depth $T_c = 2.5 \times 10^6$ K and the dashed curves for $T_c = 4.5 \times 10^6$ K. Same notation as in Fig. 2.

ΔR_c , namely $\Delta R_c / \langle C_s \rangle$, is larger for the long period models. However, when one considers it relative to the period, one finds a much larger contribution for the envelope models with the smaller periods. For example, from the figure we infer that in the 7.0 d model a small change of 10^{10} cm in R_c , i.e. $\Delta R_c / R_* \sim 0.003$, would cause an increase of P_0 by ~ 0.005 d, i.e. a $\sim 0.07\%$ relative period change. In the 0.5 d model the same change of R_c , i.e. $\Delta R_c / R_* \sim 0.02$, would cause a considerable $\sim 1\%$ change in the period.

Fortunately, an accuracy in the pulsation periods of a percent or less is generally obtainable and sufficient for most purposes, so that envelope models are therefore quite adequate. Here, however, one puts higher demands on the Cepheid models, because one wants to extract a Cepheid's metallicity from its location in the Petersen diagrams, for which a relative accuracy of $\sim 10^{-3}$ is necessary.

In Fig. 5 we present a composite Petersen diagram that compares Beat Cepheid models in which the depth is set at our standard $T_c = 2.5 \times 10^6$ K (solid lines) with those at $T_c = 4.5 \times 10^6$ K (dashed lines). One notices that the most unreliable models are to be found in the high Z , low period domain. Luckily, the vast majority of the observed Beat Cepheids avoid the regime in which the uncertainties are largest.

Fig. 5 shows that one would be ill advised to use envelope

models for the $\lesssim 1.0$ d stars that are found in the SMC (Marquette et al. (2007)). For these Beat Cepheids it is necessary to use models that extend deeper, if not full stellar models. This is, of course, a much harder problem, in that it requires the inclusion of nuclear burning and of the spatial composition variations that occur during stellar evolution. In other words, one needs a stellar evolution code that incorporates a linearization to compute the fully nonadiabatic pulsation frequencies with a time-dependent mixing length recipe to capture the pulsation – convection interaction.

4. CONCLUSION

We have computed the Beat Cepheid models that make up the composite Petersen diagrams as a function of metallicity Z with the Asplund, Grevesse & Sauval (2005) solar mixture. This change causes a noticeable reduction of $\sim 25 - 30\%$ in the average $\langle Z \rangle$ of the four galaxies in which a sizable number of Beat Cepheids are known. It considerably improves the agreement between the average Galactic metallicity and the Sun's metallicity over what was obtained with the GN93 abundances.

While the elemental mixture of the low Z stars is most likely not quite solar, this is the best approximation we can use in the absence of detailed determinations of chemical compositions from high resolution spectra of such stars.

Because we needed to construct Beat Cepheid models down to periods as short as 0.5 d as have been found in the SMC by Marquette et al. (2007), we have been led to reexamine the accuracy of commonly used envelope models for Cepheid stars. We find that because of the high accuracy demands of Petersen diagrams such envelope models become inadequate for short period, high Z Cepheids. Good fortune has it though that the higher Z Beat Cepheids in the Galaxy and in M33 have longer periods, and that the Beat Cepheids with smaller periods are observed in the lower metallicity Magellanic Clouds. Thus the vast majority of the known Beat

Cepheids avoid to some extent the troublesome region of uncertainty of envelope models.

We wish to thank Daniel Cordier, Robert Szabó and Peter Wood for valuable discussions. We are also very grateful to Carlos Iglesias and Forrest Rogers for making the computation of opacity tables on the OPAL homepage so convenient. Finally, we wish to thank an anonymous referee whose comments have improved this paper. This work has been supported by NSF (AST07-07972 and OISE04-17772) at UF.

REFERENCES

- Asplund, M., Grevesse, N. & Sauval, A.J. 2005, in *Cosmic Abundances as Records of Stellar Evolution and Nucleosynthesis*, ASP Conf. Ser. 336, 25.
- Alexander, D. R. & Ferguson, J. W. 1994, *ApJ*, 437, 879.
- Beaulieu, J.P., Buchler, J.R., Marquette, J.B. Hartman, J.D. & Schwarzenberg-Czerny, A., 2006, *ApJ*, 653, L101.
- Buchler, J. R., Kovács G. 1986, *ApJ*308, 661
- Buchler, J.R. & R. Szabo, R. 2007, *ApJ*660, 723 (Paper I)
- Grevesse, N. & Noels, A., 1993, in *Origin and evolution of the elements* Eds. N. Prantzos, E. Vangioni-Flam and M. Casse, Cambridge University Press, Cambridge, England, p.14.
- Iglesias, C. A. & Rogers, F.J. 1996, *ApJ*, 464, 943, <http://physci.llnl.gov/Research/OPAL/>
- Kolláth, Z., Buchler, J. R., Szabó, R. & Csubry, Z. 2002, *A&A*, 385, 932.
- Kolláth, Z., Beaulieu, J.P., Buchler, J. R. & Yecko, P., 1998, *ApJ*502, L55
- J.B. Marquette, J.P. Beaulieu, J.R. Buchler, R. Szabó, P. Tisserand, et al., *A&A*, 2007, in preparation.
- Stellingwerf R. F. 1974, *ApJ*, 192, 139
- Szabó, R. & Buchler, J.R. 2008, *ApJ*, (to be submitted)